FINAL REVIEW

MATH 4242 010, AU'14

Chapter 1

- Solving linear systems with Gaussian elimination
- LU factorization, permuted LU factorization, generalized permuted LU factorization.
- Using the above to solve linear systems
- Matrix inverses, rank: Know the definitions
- Determinants: Know Theorem 1.50 and how to use it, Prop 1.54
- Be able to discuss the different methods for solving $Ax = b$. When would you choose one over another?

Chapter 2

- Vector space definition/axioms, main examples
- Subspaces: definition, subspace criteria Prop 2.9
- Be able to find a basis of a subspace of $\mathbb{R}^n$ given in terms of linear equations. Note that this is identical to finding a basis of a kernel.
- Linear independence: definition for linear (in)dependence Theorem 2.21. How do you test if a set of vectors in $\mathbb{R}^n$ is linearly independent? How do you know if they form a basis?
- Span: definition Theorem 2.21, Prop 2.24 and 2.25
- Bases: definition, dimension, Theorem 2.29, Theorem 2.31, Lemmata 2.30 and 2.34,
- Kernel and Range: definition, Theorem 2.39, Propositions 2.41 and 2.42. Be able to find $\ker(A)$ and $\operatorname{rng}(A)$ and identify bases for them.
- The principal of superposition
- The fundamental theorem of linear algebra: Know how it connects the kernel, range, and rank of A and A^T.

Chapter 3

- Inner products
  - Know the definition
  - Know the standard examples: dot products, weighted dot products, the $L^2([a, b])$ inner product.
  - The Cauchy-Schwarz inequality and the triangle inequality
  - Know the classification of inner products on $\mathbb{R}^n$, Theorem 3.21
- Norms
  - Know the definition
  - Know the standard examples: L^p and L^∞ norms on $\mathbb{R}^n$, the L^p and L^∞ norms on the space of continuous functions, $C^0([a, b])$. For which p can we define the L^p norm?
- Positive definite matrices
  - Know the definition
  - Know how to check if a matrix is positive definite: Theorem 3.37. By row reducing from left to right, using ERO1, if a number less than or equal to 0 appears on the

diagonal, then the matrix is not positive definite. Otherwise the matrix is positive definite.

Chapter 4

- Minimizing quadratic polynomials
  - Know how to write a quadratic polynomial (e.g. $p(\mathbf{x}) = x_1^2 - x_1x_2 + 2x_2^2 + 4x_1 - 5$) into the matrix form $\mathbf{x}^T K\mathbf{x} - 2\mathbf{x}^T\mathbf{f} + c$.
  - Know how to tell if $p(\mathbf{x})$ has a global minimum and how to find it, Theorem 4.1.
- The nearest point problem
  - Be able to solve a nearest point problem in $\mathbb{R}^m$. In other words, given a positive definite matrix C defining an inner product, a subspace V, and a point $b \in \mathbb{R}^m$, be able to find the point $v^* \in V$ minimizing the associated norm $||v - b||$.
  - Remember, that a key step in this process is to choose a basis for V. That means that if V is n dimensional, the basis you choose should have n vectors.
- Least Squares
  - Be able to solve a least squares problem for a matrix A with $\ker(A) = \{0\}$.
  - Know how to modify this approach when $\ker(A)$ is allowed to be arbitrary.
- Data Fitting
  - Know how to fit a linear or quadratic polynomial to data.
  - In the case that the degree is one less than the number of data points, the points can be fit exactly; this is called "interpolating." Put another way, the interpolating polynomial of a set of data points is the unique polynomial with degree 1 less than the number of points which fits the points exactly.
  - For general function fitting, be able to do a problem like 4.4.33 or 4.4.35.

Chapter 5

- Orthogonality
  - Know the definition of orthogonal and orthonormal bases
  - Know how to decompose a vector in terms of an orthonormal/orthogonal basis, Theorems 5.7 and 5.9.
- The Gram-Schmidt process
  - Know how to perform the Gram-Schmidt process to turn a basis into an orthonormal basis.
  - What does it mean if the Gram-Schmidt process results in a 0 vector at some stage? What does it mean if it does not?
- The discrete Fourier Transform
  - Complex numbers and arithmetic, complex exponential, complex n^{th} roots of 1, the complex space $\mathbb{C}^n$
  - The relevant terms and definitions: the sample vector $\mathbf{f}$, the sampled exponentials $\omega_0, \ldots, \omega_{n-1}$, the averaged dot product
  - The standard Fourier transform, the low frequency alternative, the noise reducing alternative
  - The formula for iteratively reducing and reconstructing the Fourier coefficients in the Fast Fourier Transform

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- Know how to define a transition matrix from a graph of the internet and a damping factor d