MIDTERM 2 REVIEW

MATH 4242 010, AU'14

Previous Material

- Solving Linear systems
- Finding bases for $\ker(A)$ or $\operatorname{rng}(A)$.

Chapter 3

- Inner products
  - Know the definition
  - Know the standard examples: dot products, weighted dot products, the $L^2([a,b])$ inner product.
  - The Cauchy-Schwarz inequality and the triangle inequality
  - Know the classification of inner products on $\mathbb{R}^n$, Theorem 3.21
- Norms
  - Know the definition
  - Know the standard examples: L^p and L^∞ norms on $\mathbb{R}^n$, the L^p and L^∞ norms on the space of continuous functions, $C^0([a,b])$. For which p can we define the L^p norm?
- Positive definite matrices
  - Know the definition
  - Know how to check if a matrix is positive definite: Theorem 3.37

Chapter 4

- Minimizing quadratic polynomials
  - Know how to write a quadratic polynomial (e.g. $p(\mathbf{x}) = x_1^2 - x_1x_2 + 2x_2^2 + 4x_1 - 5$) into the matrix form $\mathbf{x}^T K\mathbf{x} - 2\mathbf{x}^T\mathbf{f} + c$.
  - Know how to tell if $p(\mathbf{x})$ has a global minimum and how to find it, Theorem 4.1.
- The nearest point problem
  - Be able to solve a nearest point problem in $\mathbb{R}^m$. In other words, given a positive definite matrix C defining an inner product, a subspace V, and a point $b \in \mathbb{R}^m$, be able to find the point $v^* \in V$ minimizing the associated norm $||v - b||$.
  - Remember, that a key step in this process is to choose a basis for V. That means that if V is n dimensional, the basis you choose should have n vectors. This was by far the biggest mistake in the quiz.
  - For example: If V is a one dimensional subspace of $\mathbb{R}^3$, you need one basis vector (not three). If V is a 2 dimensional subspace of $\mathbb{R}^4$, you need two basis vectors (not 4).
- Least Squares
  - Be able to solve a least squares problem for a matrix A with $\ker(A) = \{0\}$.
  - Know how to modify this approach when $\ker(A)$ is allowed to be arbitrary.
- Data Fitting
  - Know how to fit a linear or quadratic polynomial to data.

  - In the case that the degree is one less than the number of data points, the points can be fit exactly; this is called "interpolating." Put another way, the interpolating polynomial of a set of data points is the unique polynomial with degree 1 less than the number of points which fits the points exactly.
  - For general function fitting, be able to do a problem like 4.4.33 or 4.4.35.