QUIZ 11

MATH 4242 010, AU'14

Please write your **name on the top left** and show all work legibly.

The Discrete Fourier Transform

Problem 1. Draw the unit circle in the complex plane and label the 3^{rd} roots of 1, $\zeta_3^0, \zeta_3, \zeta_3^2$. Give expressions for the third roots of 1 without any numerical approximation (hint: these result in special angles). Use this to write the basis $\omega_0, \omega_1, \omega_2$.

Problem 2. Verify that, when $n = 3$, the basis $\omega_0, \omega_1, \omega_2$ is orthonormal under the averaged dot product. Verify as well that $\omega_{-1} = \omega_2$.

Problem 3. Given the sampled vector $\mathbf{f} = (1, 2, 3)$, find the Fourier coefficients c_0, c_1, c_2. Explain why $c_{-1} = c_2$. Write the standard trigonometric interpolant and the low frequency trigonmetric interpolant and in terms of sines and cosines, and simplify as much as possible.

Problem 4. Given the coefficients $c_0 = c_1 = 1, c_2 = 2$, write the sampled data $\mathbf{f}$. (Hint: remember, we chose our c_k so that $\mathbf{f} = \sum_k c_k \omega_k$.)

The Fast Fourier Transform

Problem 5. Let $\mathbf{f} = (1, 2, -1, 0)$. Notice that $n = 4 = 2^2$.

- Break the sample vector $\mathbf{f}$ into two sample vectors of length two, $\mathbf{f}^{\text{even}}$ and $\mathbf{f}^{\text{odd}}$.
- Repeat the above step on each of the two resulting sample vectors.
- Now we are down to 4 sample vectors of length one. In this case, there is only one Fourier coefficient c_0. In this case, $\omega_0 = (1)$ and so in each of the four cases, c_0 is equal to the only term in the sample vector.
- Now we reconstruct the higher sample vectors. Using the equation that

  $$c_k = (c_k^{\text{even}} + c_k^{\text{odd}} e^{2\pi i k/2^r})/2$$

  gives us the sample values at level r, iterate upward to reconstruct the fourier coefficients (Hint: see the extra Fourier examples notes for an example of this).

Google page rank

Problem 6. Given an internet with four websites labeled 1 to 4 and links $1 \to 4, 1 \to 3, 2 \to 3, 2 \to 4, 3 \to 1, 3 \to 2, 3 \to 4, 4 \to 2$ (where $a \to b$ means there is a link from a to b), draw the resulting digraph. Write the transition matrix T with a damping factor of $d = 1/2$.

Problem 7. For the above internet, we now estimate the page rank of each website. Recall that if we set $P_0 = (1, 1, 1, 1)$, the page rank as of website i is defined as the ith entry of $P_\infty = \lim_{\ell \to \infty} T^\ell P_0$. Compute T^2, $T^4 = (T^2)^2$, and $T^8 = (T^4)^2$. Use $T^8 P_0$. Use $T^8 P_0$ to approximate the page rank of each website.