

QUIZ 9

MATH 4242 010, AU'14

Please write your **name on the top left** and show all work legibly.

Problem 1. Find the line which best fits the data points

t_i		1	2	3
y_i		1	-3	-6

$$\text{Set } A = \begin{pmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} 1 \\ -3 \\ -6 \end{pmatrix}. \quad \text{Then } A^T A = \begin{pmatrix} 3 & 6 \\ 6 & 14 \end{pmatrix},$$

$$A^T \vec{y} = \begin{pmatrix} -8 \\ -23 \end{pmatrix}. \quad \text{Solving } A^T A \mathbf{x} = A^T \vec{y} \text{ yields } \left(\mathbf{x} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \right)$$

$$\left(\begin{array}{cc|c} 3 & 6 & -8 \\ 6 & 14 & -23 \end{array} \right) \xrightarrow{-2r_1} \left(\begin{array}{cc|c} 3 & 6 & -8 \\ 0 & 2 & -7 \end{array} \right) \rightsquigarrow 2\alpha_1 = -7 \Rightarrow \alpha_1 = -\frac{7}{2}$$

$$-23 + 16 = -7$$

$$3\alpha_0 = -6\alpha_1 - 8$$

$$= -6\left(-\frac{7}{2}\right) - 8 = 21 - 8 = 13$$

$$\Rightarrow \alpha_0 = \frac{13}{3}.$$

Thus, the least squares line of best fit is

$$\boxed{y(t) = \alpha_0 + \alpha_1 t = \frac{13}{3} + -\frac{7}{2}t.}$$