QUIZ 8

MATH 4242 010, AU'14

Please write your **name on the top left** and show all work legibly.

Problem 1. Let $C = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$. Define an inner product on $\mathbb{R}^2$ by $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T C \mathbf{y}$ and let $|| \cdot ||$ be its associated norm. Let $V = \{\mathbf{x} \in \mathbb{R}^2 : x_1 - 2x_2 = 0\}$. According to this inner product, what is the nearest point in V to $\mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$?

First, we note that $V = \ker(\begin{pmatrix} 1 - 2 \end{pmatrix})$. Hence, $\dim(V) = 1$ by the rank nullity theorem. Thus, we choose a basis for V consisting of $v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. So we form the matrix of basis vectors $A = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and set

$$K = A^T C A = \begin{pmatrix} 2 & 1 \end{pmatrix} C \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \end{pmatrix}, \quad \mathbf{f} = A^T C \mathbf{b} = \begin{pmatrix} 5 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 15 \end{pmatrix}.$$

The minimum $\mathbf{x}^*$ is given by solving $K\mathbf{x}^* = \mathbf{f}$. This equation is $10x_1^* = 15$, so $x_1^* = 3/2$. We conclude that $\mathbf{v}^* = x_1 \mathbf{v}_1 = 3/2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3/2 \end{pmatrix}$ is the nearest point to $\mathbf{b}$.