QUIZ 6 7

MATH 4242 010, AU'14

Please write your **name on the top left** and show all work legibly.

Problem 1. Let $K = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}$.

(a) Is K positive definite? How do you know?

(b) Does the formula $\langle x, y \rangle = x^T K y$ form an inner product on $\mathbb{R}^3$?

(c) Let $f = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$. Does the quadratic function $q(x) = x^T K x - 2x^T f + 2$ have a minimum? What is the minimizer x^*? (You do not need to calculate the minimum value)

(a) By Theorem 3.37, $K > 0$ if and only if it is regular with all positive pivots. We row reduce to find

$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \overset{r_3 + r_1}{\sim} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, and hence $K > 0$.

(b) Because K is symmetric & $K > 0$, $\langle x, y \rangle = x^T K y$ forms an inner product on $\mathbb{R}^3$.

(c) The quadratic function $q(x)$ has a minimum because $K > 0$. Moreover, by Theorem 4.1, the minimizer x^* is the solution of $Kx^* = f$. We solve

$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 \\ -1 & 0 & 3 & 0 \end{array}\right) \overset{r_3 + r_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 2 & 0 \end{array}\right) \Rightarrow x_1^* = x_3^* = 0 \quad x_2^* = -1$