**QUIZ 6**

MATH 4242 010, AU'14

Please write your **name on the top left** and show all work legibly.

**Problem 1.** Consider the inner product space $L^2([0,1])$. Let $p_1(x) = x$.
(a) What is $||p_1||$?
(b) Find a degree 1 polynomial $p_2$ which is orthogonal to $p_1$ and has norm 1 in $L^2([0,1])$.

(a) $\|p_1\| = \left( \int_0^1 (x)^2 \, dx \right)^{1/2} = \left( \frac{x^3}{3} \Big|_0^1 \right)^{1/2} = \boxed{\frac{1}{\sqrt{3}}}$

(b) $\langle p_2, p_1 \rangle = \int_0^1 (ax+b)(x) \, dx = \int_0^1 ax^2 + bx = a\frac{x^3}{3} + b\frac{x^2}{2} \Big|_0^1$

Set $p_2(x) = ax+b$

$= \frac{a}{3} + \frac{b}{2}$.

$\langle p_2, p_1 \rangle = 0 \Rightarrow \frac{a}{3} + \frac{b}{2} = 0 \Rightarrow a = -\frac{3}{2} b$

$1 = \|p_2\|^2 = \int_0^1 (ax+b)^2 = \int_0^1 a^2x^2 + 2abx + b^2 =$

$\frac{a^2}{3} + \frac{2}{4} ab + b^2 = \frac{\frac{9}{4} b^2}{3} + -\frac{3}{2} b^2 + b^2$

$= \frac{1}{4} b^2$

$\Rightarrow 4 = b^2 \Rightarrow b = \pm 2$. Choose $b=2$, $\Rightarrow a = -3$.

$\boxed{p_2 = -3x + 2}$