

QUIZ 3

MATH 4242 010, AU'14

Please write your **name on the top left** and show all work legibly.

Problem 1. Let $A = \begin{pmatrix} 1 & -2 & 0 & 3 \\ 2 & -4 & 0 & 3 \\ -1 & 0 & 4 & 1 \end{pmatrix}$. Suppose you perform Gaussian elimination with Elementary row operations 1 and 2 as follows:

$$\begin{pmatrix} 1 & -2 & 0 & 3 \\ 2 & -4 & 0 & 3 \\ -1 & 0 & 4 & 1 \end{pmatrix} \begin{matrix} r_2 - 2r_1 \\ r_3 + r_1 \end{matrix} \begin{pmatrix} 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & -3 \\ 0 & -2 & 4 & 4 \end{pmatrix} (2\ 3) \begin{pmatrix} 1 & -2 & 0 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

(a) What is/are the free variable(s)?

x_3

(b) Use the steps above to compute a generalized permuted LU factorization. That is, write A in terms of $PA = LU$ where P is a permutation matrix, L is lower triangular, and U is in row echelon form.

$$P: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$L: \begin{pmatrix} 1 & & & \\ 2 & 1 & & \\ -1 & 0 & 1 & \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & \\ -1 & 1 & & \\ 2 & 0 & 1 & \end{pmatrix} \quad \text{then so}$$

$$PA = LU, \text{ where } U \text{ is } \begin{pmatrix} 1 & -2 & 0 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 0 & 0 & -3 \end{pmatrix} \text{ above.}$$

(c) What is the solution of the homogeneous equation $Ax = 0$?

$Ax = 0 \Leftrightarrow PAx = 0 \Leftrightarrow LUx = 0$. Because L is lower triangular with nonzero diagonals, L^{-1} exists. Thus,

$$LUx = 0 \Leftrightarrow Ux = L^{-1}(0) = 0. \text{ Solving } Ux = 0,$$

$$x_3 = 0, \quad -2x_2 + 4x_3 = 0, \quad x_1 - 2x_2 = 0,$$

$$x_2 = 2x_3, \quad x_1 = 4x_3$$

$$x = \begin{pmatrix} 4x_3 \\ 2x_3 \\ x_3 \\ 0 \end{pmatrix} \text{ for any } x_3 \in \mathbb{R}.$$