

QUIZ 1 2

MATH 4242 010, AU'14

Please write your **name on the top left** and show all work legibly.

Problem 1. Let $A = \begin{pmatrix} 4 & -4 & 2 \\ -3 & 3 & 1 \\ -3 & 1 & -2 \end{pmatrix}$. Find a $PA = LU$ factorization of A and use it to solve

$Ax = b$, where $b = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$.

$$\begin{pmatrix} 4 & -4 & 2 \\ -3 & 3 & 1 \\ -3 & 1 & -2 \end{pmatrix} \xrightarrow{(12)} \begin{pmatrix} -3 & 3 & 1 \\ 4 & -4 & 2 \\ -3 & 1 & -2 \end{pmatrix} \xrightarrow{\substack{r_2 + \frac{4}{3}r_1 \\ r_3 - r_1}} \begin{pmatrix} -3 & 3 & 1 \\ 0 & 0 & \frac{10}{3} \\ 0 & -2 & -3 \end{pmatrix} \xrightarrow{(23)} \begin{pmatrix} -3 & 3 & 1 \\ 0 & -2 & -3 \\ 0 & 0 & \frac{10}{3} \end{pmatrix}$$

$$P: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$L: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Problem 2. Here's a problem.

$$Ax = b \Rightarrow PAx = Pb = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}. \quad PA = LU \Rightarrow$$

$$LUx = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}. \quad \text{Solve 2 systems, } Lc = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, \quad Ux = c.$$

$$c_1 = 3$$

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$$c_1 + c_2 = -5 \Rightarrow$$

$$c_2 = -8$$

$$-\frac{4}{3}c_1 + c_3 = 1$$

$$c_3 = 5$$

$$\frac{10}{3}x_3 = 5$$

\Rightarrow

$$x_3 = \frac{3}{2}$$

$$-2x_2 - 3x_3 = -8 \Rightarrow$$

$$x_2 = 4 - \frac{3}{2}x_3 = 4 - \frac{9}{4} = \frac{7}{4}$$

$$-3x_1 + 3x_2 + x_3 = 3 \Rightarrow$$

$$x_1 = -1 + x_2 + \frac{1}{3}x_3 = -1 + \frac{7}{4} + \frac{1}{2} = \frac{5}{4}$$